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Complex Sasakian Contact Metric Manifolds

Pawan Mehrda[1] and Shankar Lal[2]

[1]*Department of Mathematics, H.N.B. Garhwal University (A Central University)*
[2]*S.R.T. Campus Badshahithaul-249199, Tehri Garhwal, Uttarakhand.*

*Corresponding Author Email: pawanve0786@gmail.com

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Abstract: We study of complex sasakian manifold which is a type of normal complex contact metric manifold and study of complex η-Einstein sasakian manifold [Vanli and Unal, 2017]. Inan Unal and A. Turgut Vanli obtained curvature properties of complex Sasakian manifold [Vanli and Unal, 2019]. We define some relations between curvature tensors of complex Sasakian manifold and obtained some useful results.
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Introduction

Kobayashi first introduced the complex contact manifolds [Kobayashi, 1959]. Korkmaz gives the notion of normality conditions for complex almost contact metric manifolds [Jeffrey L. M. 2009]. Ishihara and Konishi sasakian manifold which is called IK-normality [Ishihara and Konishi, 1980]. IK-normal complex contact metric manifolds, normal complex contact metric manifolds and complex sasakian manifolds are three notions of the Riemannian geometry of complex almost contact metric manifolds.

Aysel Turgut and Inan Unal defined the complex sasakian manifold and curvature properties. Also defined the symmetry conditions for complex sasakian manifolds with related to concircular curvature tensor, conformal curvature tensor, conharmonic curvature tensor and projective curvature tensor.

In this present paper we studied complex η-Einstein sasakian manifold and obtain some result. We study on concircular curvature tensor K in a sasakian manifold and we obtain some result abouts concircular curvature tensor.

Preliminaries

Definitions 2.1. An open covering is covered by coordinate neighborhoods $\mathbb{C} = \{\mathcal{O}_i\}$ on odd dimensional $(2k+1)$ complex manifold. If there exist an analytic 1-form ω_i, then

$$\omega_i \wedge (d\omega_j)^k \neq 0 \quad \text{in } \mathcal{O}_i$$

and, if $\mathcal{O}_i \cap \mathcal{O}_j \neq \emptyset$, then there is an analytic function f_{ij} in $\mathcal{O}_i \cap \mathcal{O}_j$ such that

$$\omega_i = f_{ij}\omega_j \quad \text{in } \mathcal{O}_i \cap \mathcal{O}_j.$$

Then this set $\{(\omega_i, \mathcal{O}_i) | \mathcal{O}_i \in \mathbb{C}\}$ is called complex contact manifold.

On each $\mathcal{O}_i$, we define

$$\mathcal{H}_i = \{\mathcal{X}_P : \omega_i(\mathcal{X}_P) = 0, \mathcal{X}_P \in T_P M\}.$$

This distribution is analytic non-integrable distribution on M.

We know that an analytic vector field is ζ_i is given by

$$\omega_i(\zeta_i) = 1.$$

Also, by the complex line, we have

$$E_i = \text{Span}\{\zeta_i\}.$$

Definition 2.2. A Hermitian metric g and J denote the complex structure on complex manifold M.Then M is a complex almost contact metric manifold, if there exist an open covering $\mathbb{C} = \{\mathcal{O}_i\}$ on M,

Therefore

(1) If on each $\mathcal{O}_i$, there exist an analytic $1-$ form u and $v = u \circ J$ and we take dual vector fields $\mathcal{U}$ and $\mathcal{V} = -J\mathcal{U}$ and there $(1,1)$ tensor fields G_α and $H_\alpha = G_\alpha J$, then

$$H_\alpha{}^2 = G_\alpha{}^2 = -I + u \otimes \mathcal{U} + v \otimes \mathcal{V}$$

$$g(\mathcal{X}, G_\alpha \mathcal{Y}) = -g(G_\alpha \mathcal{X}, \mathcal{Y})$$

$$G_\alpha J = -JG_\alpha, \quad G_\alpha \mathcal{U} = 0$$

$$g(\mathcal{U}, \mathcal{U}) = 1. \qquad \text{...(2.1)}$$

(2) If $\mathcal{O}_i \cap \mathcal{O}_j \neq \emptyset$, then we have

$$u' = Cu - Dv, \quad v' = Du + Cv$$

$$G' = CG_\alpha - DH_\alpha, \quad H' = DG_\alpha + CH_\alpha$$

For some functions C, D defined on $\mathcal{O}_i \cap \mathcal{O}_j$ with $C^2 + D^2 = 1$.

We know that a complex almost contact metric manifold M, holds the following identities:

$$H_\alpha G_\alpha = -G_\alpha H_\alpha = J + u \otimes \mathcal{V} - v \otimes \mathcal{U}$$

$$g(H_\alpha \mathcal{X}, \mathcal{Y}) = -g(\mathcal{X}, H_\alpha \mathcal{Y})$$

$$JH_\alpha = -H_\alpha J = G_\alpha, \quad G_\alpha \mathcal{V} = H_\alpha \mathcal{U} = H_\alpha \mathcal{V} = 0$$

$$uG_\alpha = vG_\alpha = uH_\alpha = vH_\alpha = 0$$

$$J\mathcal{V} = \mathcal{U}, \; g(\mathcal{U}, \mathcal{V}) = 0.$$

A complex contact manifold contains a complex almost contact metric structure. Let $\omega = u + iv$ be a local contact from. So, the local tensor field G_α, H_α and du, dv are organized by

$$du(\mathcal{X}, \mathcal{Y}) = \widehat{G}_\alpha(\mathcal{X}, \mathcal{Y}) + (\sigma \wedge v)(\mathcal{X}, \mathcal{Y}),$$

$d v(\mathcal{X},\mathcal{Y}) = \widehat{H}_\alpha(\mathcal{X},\mathcal{Y}) - (\sigma \vee u)(\mathcal{X},\mathcal{Y}),$

where σ is 1 – form and $\widehat{G}_\alpha(\mathcal{X},\mathcal{Y}) = g(\mathcal{X},\widehat{G}_\alpha\mathcal{Y})$ and $\widehat{H}_\alpha(\mathcal{X},\mathcal{Y}) = g(\mathcal{X},\widehat{H}_\alpha\mathcal{Y})$ with $\sigma(\mathcal{X}) = g(\nabla_{\mathcal{X}}\mathcal{U})$.

Sasakian manifolds has an important role in real contact geometry and complex contact geometry.

Definition 2.3.([Korkmaz, 2000]) Korkmaz gave the definition of complex contact metric manifold M. M is said to be normal, if

(1) $\mathcal{S}(\mathcal{X},\mathcal{Y}) = \mathcal{T}(\mathcal{X},\mathcal{Y}) = 0$, for all $\mathcal{X},\mathcal{Y}$ are in $\mathbb{H}$. and

(2) $\mathcal{S}(\mathcal{U},\mathcal{X}) = \mathcal{T}(\mathcal{V},\mathcal{X}) = 0$, for all $\mathcal{X}$,

Here $\mathcal{S}$ and $\mathcal{T}$ are $(1,2)$ Ishihara and Konishi tensors [Ishihara and Konishi, 1980] defined on M.

$$\mathcal{S}(\mathcal{X},\mathcal{Y}) = [G,G](\mathcal{X},\mathcal{Y}) + 2(v(\mathcal{Y})H\mathcal{X} - v(\mathcal{X})H\mathcal{Y}) + 2g(\mathcal{X},G\mathcal{Y})\mathcal{U} - 2g(\mathcal{X},H\mathcal{Y})\mathcal{V} - \sigma(G\mathcal{X})H\mathcal{Y} + \sigma(G\mathcal{Y})H\mathcal{X} + \sigma(\mathcal{X})GH\mathcal{Y} - \sigma(\mathcal{Y})GH\mathcal{X},$$

$$\mathcal{T}(\mathcal{X},\mathcal{Y}) = [H,H](\mathcal{X},\mathcal{Y}) + 2(u(\mathcal{Y})G\mathcal{X} - u(\mathcal{X})G\mathcal{Y}) + 2g(\mathcal{X},H\mathcal{Y})\mathcal{V} - 2g(\mathcal{X},G\mathcal{Y})\mathcal{U} + \sigma(H\mathcal{X})G\mathcal{Y} - \sigma(H\mathcal{Y})G\mathcal{X} - \sigma(\mathcal{X})HG\mathcal{Y} + \sigma(\mathcal{Y})HG\mathcal{X}.$$

where $[G,G](\mathcal{X},\mathcal{Y}) = (\nabla_{G\mathcal{X}}G)\mathcal{Y} - (\nabla_{G\mathcal{Y}}G)\mathcal{X} - G(\nabla_{\mathcal{X}}G)\mathcal{Y} + G(\nabla_{\mathcal{Y}}G)\mathcal{X}$ is the Nijenhuis tensor of G.

Definition 2.4. Let $(M, G_\alpha, H_\alpha, J, \mathcal{U}, \mathcal{V}, u, v)$ be a normal complex contact manifold and g be Hermitian metric and $\omega = u - iv$ is globally defined. Then $\widehat{G}_\alpha$ and $\widehat{H}_\alpha$ 2 – forms are defined by

$\widehat{G}_\alpha(\mathcal{X},\mathcal{Y}) = d u(\mathcal{X},\mathcal{Y})$ and $\widehat{H}_\alpha = d v(\mathcal{X},\mathcal{Y})$ where $\mathcal{X}$ and $\mathcal{Y}$ are vector fields on M.

Theorem 2.1.([Vanli and Unal, 2019]) If M is a normal complex contact metric manifold, then it is a complex sasakian manifold if and only if

$$(\nabla_{\mathcal{X}}G_\alpha) = -2v(\mathcal{X},\mathcal{Y})HG\mathcal{Y} - u(\mathcal{Y})\mathcal{X} - v(\mathcal{Y})J\mathcal{X} + g(\mathcal{X},\mathcal{Y})\mathcal{U} + g(J\mathcal{X},\mathcal{Y})\mathcal{V},$$

$$(\nabla_{\mathcal{X}}H_\alpha) = -2u(\mathcal{X})HG\mathcal{Y} + u(\mathcal{Y})J\mathcal{X} - v(\mathcal{Y})\mathcal{X} - g(J\mathcal{X},\mathcal{Y})\mathcal{U} + g(\mathcal{X},\mathcal{Y})\mathcal{V}.$$

and

$$(\nabla_{\mathcal{X}}J)\mathcal{Y} = -2u(\mathcal{X})H_\alpha\mathcal{Y} + 2v(\mathcal{X})G_\alpha\mathcal{Y}.$$

Now, for complex Sasakian manifold we get

$$\nabla_{\mathcal{X}}\mathcal{U} = -G_\alpha\mathcal{X}, \quad \nabla_{\mathcal{X}}\mathcal{V} = -G_\alpha\mathcal{X}.$$

If M be a complex sasakian manifold. Then we have following curvature properties of complex sasakian manifolds

$$R(\mathcal{X},\mathcal{Y})\mathcal{U} = \mathcal{X} + u(\mathcal{X})\mathcal{U} + v(\mathcal{X})\mathcal{V}, \quad ...(2.2)$$

$$R(\mathcal{X},\mathcal{Y})\mathcal{V} = \mathcal{X} - u(\mathcal{X})\mathcal{U} - v(\mathcal{X})\mathcal{V}, \quad ...(2.3)$$

$$R(\mathcal{X},\mathcal{U})\mathcal{V} = -3J\mathcal{X} - 3u(\mathcal{X})\mathcal{V} + 3v(\mathcal{X})\mathcal{U}, \quad ...(2.4)$$

$$R(\mathcal{X},\mathcal{V})\mathcal{U} = 0, \quad ...(2.5)$$

$$R(\mathcal{X},\mathcal{Y})\mathcal{U} = u(\mathcal{X})J\mathcal{Y} - v(\mathcal{Y})J\mathcal{X} + u(\mathcal{Y})\mathcal{X} - u(\mathcal{X})\mathcal{Y} + 2v(\mathcal{X})u(\mathcal{Y})\mathcal{V} - 2v(\mathcal{Y})u(\mathcal{X})\mathcal{V} - 2g(J\mathcal{X},\mathcal{Y})V, \quad ...(2.6)$$

$$R(\mathcal{X},\mathcal{Y})\mathcal{V} = 3u(\mathcal{X})J\mathcal{Y} - 3u(\mathcal{Y})J\mathcal{X} + 2u(\mathcal{Y})v(\mathcal{X})\mathcal{U} - 2u(\mathcal{X})v(\mathcal{Y})\mathcal{U} + v(\mathcal{Y})\mathcal{X} - v(\mathcal{X})\mathcal{Y} + 2g(J\mathcal{X},\mathcal{Y})\mathcal{U}, \quad ...(2.7)$$

$$R(\mathcal{U},\mathcal{V})\mathcal{X} = J\mathcal{X} + u(\mathcal{X})\mathcal{V} - v(\mathcal{X})\mathcal{U}, \quad ...(2.8)$$

$$R(\mathcal{X},\mathcal{V})\mathcal{Y} = 2u(\mathcal{Y})u(\mathcal{X})\mathcal{V} + 3u(\mathcal{Y})J\mathcal{X} + 3g(J\mathcal{Y},\mathcal{X})\mathcal{U} - 2v(\mathcal{Y})u(\mathcal{X})\mathcal{U} - g(\mathcal{Y},\mathcal{X})\mathcal{V} + v(\mathcal{Y})\mathcal{X} - 2u(\mathcal{X})J\mathcal{Y}, \quad ...(2.9)$$

$$R(\mathcal{X},\mathcal{U})\mathcal{Y} = 2u(\mathcal{Y})v(\mathcal{X})\mathcal{U} - 2v(\mathcal{Y})v(\mathcal{X})\mathcal{U} - g(\mathcal{Y},\mathcal{X})\mathcal{U} + u(\mathcal{Y})\mathcal{X} + g(J\mathcal{Y},\mathcal{X})\mathcal{V}. \quad ...(2.10)$$

where $\mathcal{U}$ and $\mathcal{V}$ are vertical vector fields and $\mathcal{V} = -J\mathcal{U}$ then we have

$R(\mathcal{U},\mathcal{V})\mathcal{V} = R(\mathcal{V},\mathcal{U})\mathcal{U} = 0$

$R(\mathcal{U},\mathcal{V},\mathcal{V},\mathcal{U}) = R(\mathcal{V},\mathcal{U},\mathcal{U},\mathcal{V}) = 0$

Ricci curvature tensor of a complex sasakain manifold presented by Turgut Vanli and Unal [Vanli and Unal,2017, 2019, 2020] as

$$s(\mathcal{U},\mathcal{U}) = s(\mathcal{V},\mathcal{V}) = 4k, \quad s(\mathcal{U},\mathcal{V}) = 0, \quad ...(2.11)$$

$s(\mathcal{X},\mathcal{U}) = 4k\,u(\mathcal{X}), \quad s(\mathcal{X},\mathcal{V}) = 4k\,v(\mathcal{X}),$

$s(\mathcal{X},\mathcal{Y}) = s(G\mathcal{X},G\mathcal{Y}) + 4k\big(u(\mathcal{X})u(\mathcal{Y}) + v(\mathcal{X})v(\mathcal{Y})\big),$

$s(\mathcal{X},\mathcal{Y}) = s(H\mathcal{X},H\mathcal{Y}) + 4k\big(u(\mathcal{X})u(\mathcal{Y}) + v(\mathcal{X})v(\mathcal{Y})\big),$

where $\mathcal{X},\mathcal{Y} \in \Gamma(TM)$.

Complex η -Einstein Sasakain manifold

Let $(M, G_\alpha, H_\alpha, \mathcal{U}, \mathcal{V}, u, v, g)$ be a complex sasakian manifold and $\omega = u - iv$. If α and β are smooth functions on M, if the Ricci tensor, satisfies

$$s = \alpha g + \beta(u \otimes \mathcal{U} + v \otimes \mathcal{V}). \quad ...(3.1)$$

Then M is called complex η –Einstein sasakian manifold.

On a complex η –Einstein sasakian manifold, we derive the following property

$\alpha + \beta = 4k$

If $\beta = 0$ then the manifold M is η – Einstein.

Corollary 3.1. If an odd dimensional complex sasakian manifold is Einstein then $s(\mathcal{X},\mathcal{Y}) = 4k\,g(\mathcal{X},\mathcal{Y})$ also, we have

$Q\mathcal{X} = \alpha\mathcal{X} + \beta(u(\mathcal{X})\mathcal{U} + v(\mathcal{X})\mathcal{V}),$

where $s(\mathcal{X},\mathcal{Y}) = g(Q\mathcal{X},\mathcal{Y})$ and Q is Ricci operator.

If X_0, Y_0 are two horizontal vector fields, on complex sasakian manifold M, then from (3.1), we get $s(\mathcal{X}_0,\mathcal{Y}_0) = 4kg(\mathcal{X}_0,\mathcal{Y}_0)$.

The curvature tensors on a complex sasakian manifold admits the following properties

$$\mathcal{P}(\mathcal{V},\mathcal{W})\mathcal{V} = -\frac{1}{4k+1}\mathcal{V}, \quad ...(3.2)$$

$$\mathcal{P}(\mathcal{Y},\mathcal{W})\mathcal{V} = 2g(J\mathcal{Y},\mathcal{W})\mathcal{V}, \quad ...(3.3)$$

$$\mathcal{P}(\mathcal{V},Q\mathcal{V})\mathcal{V} = -\frac{1}{4k+1}Q\mathcal{V} - \frac{1}{4k+1}s(Q\mathcal{V},\mathcal{V}), \quad ...(3.4)$$

$$\mathcal{P}(\mathcal{V},\mathcal{U})\mathcal{V} = \frac{4k}{4k+1}\mathcal{U}, \quad ...(3.5)$$

$$\mathcal{P}(\mathcal{V},\mathcal{W})\mathcal{Y} = g(J\mathcal{Y},\mathcal{W})\mathcal{U} + g(\mathcal{Y},\mathcal{W})\mathcal{V} - \frac{1}{4k+1}s(\mathcal{W},\mathcal{Y})\mathcal{V}, \quad ...(3.6)$$

Proposition 3.1**.** If M is a complex sasakian manifold then the projective curvature tensor $\mathcal{P}$.

It satisfies the following identities:

(i) $\mathcal{P}(\mathcal{X},\mathcal{Y})Z + \mathcal{P}(\mathcal{Y},\mathcal{Z})\mathcal{X} + \mathcal{P}(\mathcal{Z},\mathcal{X})\mathcal{Y} = 0,$

(ii) $\mathcal{P}(\mathcal{X},\mathcal{Y}) = -\mathcal{P}(\mathcal{Y},\mathcal{X})$.

Theorem 3.1. Let vector field $\mathcal{Y}$ on complex sasakian manifold M which satisfy $R(\mathcal{V},\mathcal{Y})\mathcal{P} = 0$, then M is a complex η-Einstein sasakian manifold.

Proof. On a complex sasakian manifold M, take a $(1,3)$ tensor field. We have

$$(T_1(\mathcal{X},\mathcal{Y}).T_2)(\mathcal{T},\mathcal{V})\mathcal{W} = T_1(\mathcal{X},\mathcal{Y})T_2(\mathcal{T},\mathcal{V})\mathcal{W} - T_2(T_1(\mathcal{X},\mathcal{Y})\mathcal{T},\mathcal{V})\mathcal{W}$$
$$- T_2(\mathcal{T},T_1(\mathcal{X},\mathcal{Y})\mathcal{V})\mathcal{W} - T_2(\mathcal{T},\mathcal{V})T_1(\mathcal{X},\mathcal{Y})\mathcal{W}. \quad ...(3.7)$$

If a complex sasakian manifold satisfies the following property for all $\mathcal{Y} = \Gamma(TM)$,

$$R(\mathcal{V},\mathcal{Y})\mathcal{P} = 0 \quad ...(3.8)$$

From equation (3.7), we get

$$(R(\mathcal{V},\mathcal{Y})\mathcal{P})(\mathcal{V},\mathcal{W})\mathcal{V} = R(\mathcal{V},\mathcal{Y})\mathcal{P}(\mathcal{V},\mathcal{W})\mathcal{V} - \mathcal{P}(R(\mathcal{V},\mathcal{Y})\mathcal{V},\mathcal{W})\mathcal{V}$$
$$- \mathcal{P}(\mathcal{V},R(\mathcal{V},\mathcal{Y})\mathcal{W})\mathcal{V} - \mathcal{P}(\mathcal{V},\mathcal{W})R(\mathcal{V},\mathcal{Y})\mathcal{V}. \quad ...(3.9)$$

Since, from equation (3.8) and (3.9),we obtain the result

$$R(\mathcal{V},\mathcal{Y})\mathcal{P}(\mathcal{V},\mathcal{W})\mathcal{V} = \mathcal{P}(R(\mathcal{V},\mathcal{Y})\mathcal{V}),\mathcal{W}) + \mathcal{P}(\mathcal{V},R(\mathcal{V},\mathcal{Y})\mathcal{W})\mathcal{V}$$
$$+ \mathcal{P}(\mathcal{V},\mathcal{W})R(\mathcal{V},\mathcal{Y})\mathcal{V} \quad (3.10)$$

In equation (3.10) if we taking inner product with $\mathcal{V}$ and using the equations $(2.3),(2.7),(2.9)$ and equations (3.2) to (3.6), we obtain the following result

$$s(\mathcal{Y},\mathcal{W}) =- 4kg(\mathcal{Y},\mathcal{W}) \quad , \qquad \mathcal{Y},\mathcal{W} \in \Gamma(\mathbb{H}).$$

$$s(\mathcal{Y},\mathcal{W}) =- 4kg(\mathcal{Y},\mathcal{W}) + 8k(u(\mathcal{Y})u(\mathcal{W}) + v(\mathcal{Y})v(\mathcal{W}),$$

where we using $\mathcal{Y} = \mathcal{Y}_0 + u(\mathcal{Y})\mathcal{U} + v(\mathcal{Y})\mathcal{V}$ and $\mathcal{W} = \mathcal{W}_0 + u(\mathcal{W})\mathcal{U} + v(\mathcal{W})\mathcal{V}$.

Therefore, the manifold M is complex η- Einstein sasakian manifold.

Now in a complex sasakian manifold, projective curvature tensor is defined as,

$$\mathcal{P}(\mathcal{X},\mathcal{Y})\mathcal{Z} - \frac{1}{4k+1}[s(\mathcal{Y},\mathcal{Z})\mathcal{X} - s(\mathcal{X},\mathcal{Z})\mathcal{Y}]. \qquad ...(3.11)$$

Theorem 3.2. ([Vanli and Unal, 2020]) Let vector field $\mathcal{Y}$ on complex sasakian manifold M which is satisfy $\mathcal{K}(V,Y)\mathcal{P} = 0$, then M is Ricci flat or a complex η-Einstein sasakian manifold.

Proposition 2. Let M be a complex sasakian manifold which is Ricci flat. Then we obtain relations between curvature tensors:

(i) $\mathcal{K}(\mathcal{X},\mathcal{Y})\mathcal{Z} = \frac{1}{4k}[(4k+1)\mathcal{P}(\mathcal{X},\mathcal{Y})\mathcal{Z} - R(\mathcal{X},\mathcal{Y})\mathcal{Z}]$

(ii) $\mathcal{C}(\mathcal{X},\mathcal{Y})\mathcal{Z} = \mathcal{K}(\mathcal{X},\mathcal{Y})\mathcal{Z} - \frac{\tau}{(4k+1)4k}[g(\mathcal{X},\mathcal{Z})\mathcal{Y} - g(\mathcal{Y},\mathcal{Z})\mathcal{X}]$,

where $\mathcal{K},\mathcal{P},\mathcal{C}$ are conharmonic curvature tensor, projective curvature tensor and conformal curvature tensor on a complex sasakian manifold.

Proposition 3.3. On a (2k+1) odd dimensional complex sasakian manifold M the concircular curvature tensor is defined by

$$K(\mathcal{X},\mathcal{Y})\mathcal{Z} = R(\mathcal{X},\mathcal{Y})\mathcal{Z} - \frac{s}{(4k+1)(4k+2)}[g(\mathcal{Y},\mathcal{Z})\mathcal{X} - g(\mathcal{X},\mathcal{Z})\mathcal{Y}],$$

where $\mathcal{X},\mathcal{Y},\mathcal{Z}$ and s are respectively vector fields on M and scalar curvature.

Now, we derive some results of concircular curvature tensor for complex sasakian manifold:

1. $K(\mathcal{X},\mathcal{U})\mathcal{U} = \left(1 - \frac{s}{(4k+1)(4k+2)}\right)\mathcal{X} + u(\mathcal{X})\mathcal{U} + v(\mathcal{X})\mathcal{V}$,
2. $K(\mathcal{X},\mathcal{Y})\mathcal{U} = R(\mathcal{X},\mathcal{Y})\mathcal{U}$,
3. $K(\mathcal{X},\mathcal{Y})\mathcal{V} = R(\mathcal{X},\mathcal{Y})\mathcal{V}$,
4. $K(\mathcal{X},\mathcal{U})\mathcal{V} = R(\mathcal{X},\mathcal{U})\mathcal{V}$,
5. $K(\mathcal{X},\mathcal{V})\mathcal{U} = R(\mathcal{X},\mathcal{V})\mathcal{U}$,
6. $K(\mathcal{U},\mathcal{V})\mathcal{X} = J\mathcal{X} + u(\mathcal{X})\mathcal{V} - v(\mathcal{X})\mathcal{U}$.

Conclusion

Complex contact manifolds have lots of applications. There are many open problems with complex contact manifolds. We investigated the relationships between curvature tensors (projective curvature tensor, conharmonic curvature tensor, and concircular curvature tensor) and the concircular curvature tensor results on complex sasakian manifolds in this paper.

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